Combinatorics of F_σ quotients

Jörg Brendle

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I plan to discuss some results on the combinatorics of quotients of the form $\mathcal{P}(\omega)/\mathcal{I}$ where $\mathcal{I}$ is an F_σ ideal on the integers. Such quotients are σ-closed and thus look rather similar to the classical structure $\mathcal{P}(\omega)/\mathrm{Fin}$. Some results which hold in the classical case can be generalized to the context of arbitrary F_σ ideals. For example, by results of Todorčević, the gap structure of $\mathcal{P}(\omega)/\mathcal{I}$ includes the one of $\mathcal{P}(\omega)/\mathrm{Fin}$, and there are (ω_1, ω_1)-Hausdorff gaps and $(\mathfrak{b}, \omega)$-Rothberger gaps. However, there may be other types of gaps as well: an example is the quotient via the ideal $\mathcal{ED}_{\mathrm{fin}}$ which contains an (ω_1, ω)-Rothberger gap. Similarly, many inequalities between, and consistency results about, cardinal invariants which hold for $\mathcal{P}(\omega)/\mathrm{Fin}$ are also true for $\mathcal{P}(\omega)/\mathcal{I}$. In fact, cardinal invariants of $\mathcal{P}(\omega)/\mathcal{I}$ are rather hard to distinguish from their classical counterparts, but there are a few consistency results.