Title: Forcing Axioms and the Continuum Hypothesis

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Abstract: It is known that there is a strongest consistent forcing axiom, namely the forcing axiom for partial orders which preserve stationary subsets of ω_1 (also known as Martin's Maximum or MM). The consistency of this axiom was established by Foreman, Magidor, and Shelah relative to the existence of a supercompact cardinal. Since MM has proved very effective at settling statements left undecided by ZFC, it is natural to ask whether there is an analogous optimal forcing axiom which is relatively consistent with the Continuum Hypothesis. One precise way to cast this question is whether there are two Π_2 sentences in the language of $(H(\omega_2), \in, \mathrm{NS}_{\omega_1})$ which are each Ω-consistent with CH but which jointly negate CH. This problem is due to Woodin, who showed that the answer is negative if one replaces CH with ZFC. We show that the answer to Woodin's problem is positive and in the process establish that there are two preservation theorems for not adding reals which can not be subsumed into a single iteration theorem. This is joint work with David Aspero and Paul Larson.